



## ***Using Graduated Sequence of Instruction to Teach Math***

### **What is the evidence base?**

This is a research-based practice for **students with disabilities** based on one methodologically sound group and three methodologically sound single-case studies across 60 students with disabilities.

This is a research-based practice for **students with learning disabilities** based on one methodologically sound group and three methodologically sound single-case studies across 48 students with learning disabilities.

This is a promising practice for **students with intellectual disability** based on one methodologically sound single-case study across one student with intellectual disability.

This is a promising practice for **students with other health impairment** based on one methodologically sound single-case study across one student with other health impairment.

### **Where is the best place to find out how to do this practice?**

The best place to find out how to implement graduated sequence of instruction (GSI) is through the following research to practice lesson plan starters:

- [Algebraic Integer Subtraction \(Maccini & Ruhl, 2000\)](#)
- [Concrete Algebra Instruction \(Cease-Cook, 2013; Witzel, 2005\)](#)
- [Representational Algebra Instruction \(Cease-Cook, 2013; Witzel, 2005\)](#)
- [Abstract Algebra Instruction \(Cease-Cook, 2013; Witzel, 2005\)](#)
- [Geometry – Perimeter \(Cass et al., 2003\)](#)
- [Geometry – Area \(Cass et al., 2003\)](#)

### **With who was it implemented?**

- Students with
  - Learning disabilities (4 studies, n=48)
  - Intellectual disability (2 studies, n=3)
  - Other health impairment (1 study, n=1)
  - Other (1 study, n=8)

- Grades ranged from 6<sup>th</sup> – 10<sup>th</sup>
- Males (n=34), females (n=26)
- Ethnicity
  - Caucasian (n=9)
  - African American (n=1)
  - Unspecified (n=50)

## **What is the practice?**

GSI has been defined as “a three-stage learning process where students learn through physical manipulation of concrete objects, followed by learning through pictorial representations of the concrete manipulations, and ending with solving problems using abstract notation” (Witzel, 2005, p. 50). Other related terms may include graduated instructional sequence, concrete-representational-abstract (CRA), or concrete-semiconcrete-abstract (CSA). Graduated sequence of instruction always includes a three-stage process for solving problems that progresses from using physical manipulatives (e.g., blocks, counters, algebra tiles), to two-dimensional representations (e.g., drawings, pictures, virtual manipulatives), and finally to abstract notation (i.e., numbers, symbols, variables).

In the studies used to establish the evidence base for using graduated sequence of instruction to teach math, interventions included:

- Graduated sequence of instruction to teach:
  - Fraction equivalence concepts (Bouck et al., 2017; Butler, Miller, Crehan, Babbit, & Pierce, 2003)
  - Perimeter and area problem solving (Cass, Cates, Smith, & Jackson, 2003)
  - Change making problem solving (Bouck, Park, & Nickell, 2017)

## **Where has it been implemented?**

- Resource classrooms (2 studies)
- Special education classroom (2 studies)

## **How does this practice relate to Common Core Standards?**

- Math, High School: Algebra, Reasoning with Equations & Inequalities
  - CCSS.MATH.CONTENT.HAS.REI.B.3 – Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters
  - CCSS.MATH.CONTENT.HSA.REI.B.4 – Solve quadratic equations in one variable.

- Math, Grade 6, Geometry
  - CCSS.MATH.CONTENT.6.G.A.1 – Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

### **How does this practice relate to the Common Career Technical Core?**

- Science, Technology, Engineering & Mathematics Career Cluster® (ST)
  - Apply engineering skills in a project that requires project management, process control and quality assurance.
- Science & Mathematics Career Pathway (ST-SM)
  - Apply science and mathematics to provide results, answers and algorithms for engineering and technological activities.
  - Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.

### **References used to establish this evidence base:**

- Bouck, E. C., Bassette, L., Shurr, J., Park, J., Kerr, J., & Whorley, A. (2017). Teaching equivalent fractions to secondary students with disabilities via the virtual-representational-abstract instructional sequence. *Journal of Special Education Technology*, 32, 220-231. doi: 10.1177/0162643417727291
- Bouck, E. C., Park, J., & Nickell, B. (2017). Using the concrete-representational-abstract approach to support students with intellectual disability to solve change-making problems. *Research in Developmental Disabilities*, 60, 24-36. doi: 10.1016/j.ridd.2016.11.006
- Butler, F. M., Miller, S. P., Crehan, K., Babbitt, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice*, 18, 99-111. doi: 10.1016/j.ridd.2016.11.006
- Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction on solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice*, 18, 112-120. doi: 10.1111/1540-5826.00067

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